

Electron random walk in ideal phonon gas. Exact dressed electron density matrix evolution equations

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Abstract. An original approach is suggested to analysis of full quantum Liouville equation for single electron (quantum particle) interacting with ideal phonon gas (harmonic boson thermostat). It is shown that under the thermodynamic limit this equation can be exactly reduced to a system of evolution equations connecting density matrix of the electron and its simplest irreducible correlations with amplitudes of one, two, three and more phonon modes. Possible application of these new equations to explanation of the electron mobility 1/f-type fluctuations in semiconductors and other media is discussed. The special case of electron in static disorder is also considered.

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1. Introduction

Here I leave for a time the problem of molecular Brownian motion and related mobility 1/f-type fluctuations in classical fluids [1, 2, 3, 4, 5, 6, 7, 8, 9] and turn to similar problems concerning random walks of electric charge carriers in conducting media and related electronic 1/f noise [10, 11, 12, 13]. Although the second subject seems much more interesting from practical point of view, the first one gave more possibilities to examine how and why the conventional kinetic approximations of statistical mechanics had lost so important phenomenon as 1/f noise. Now it is clear that the Boltzmann's conjectures are wrong, and in more correct approximations any particle of both dilute gas and dense fluid manifests unique random diffusivity and mobility (what, in essence, was predicted already in [14]). In the light of these achievements we may try now revision of kinetic models of quantum statistical mechanics, beginning, naturally, from the quasi-free electron in thermally vibrating crystal lattice. Or, in other words, electron in phonon bath (or, in generalized terminology, quantum particle in boson field).

Of course, real crystal lattices vibrate anharmonically. At that the anharmonicity gives rise to not only relaxation of any particular phonon mode but also $1/f$ -type fluctuations in rate of the relaxation [15]. These fluctuations can be transmitted to electron's relaxation due to phonons and thus to electron's mobility.

But the electron-phonon interaction by itself causes relaxation of electron's velocity and energy. Hence, - as experience of investigation of the mentioned classical systems says, - it by itself can cause also $1/f$ -type fluctuations in the rate of this relaxation and consequently in diffusivity and mobility of the electron. Therefore it is reasonable for the beginning to consider electron in harmonic lattice, that is in ideal phonon gas.

In this paper our purpose is to reveal a minimal set of relevant statistical characteristics which completely and exactly determine all effects of the electron-phonon interaction, and derive exact evolution equations for such characteristics. The results will present a necessary mathematical base for further investigations.

2. The model and the problem

Let us consider a single quantum particle interacting with a potential field formed by very many mutually non-interacting plane-wave quantized excitations. The latter will be called "phonons" while the particle "electron", although it will be spinless, or "Brownian particle" (BP), since its interaction with phonons may enforce it to a random walk, at least in the thermodynamic limit and at non-zero temperature when the phonons can serve as thermostat with infinitely large total energy.

2.1. Basic equations

The Hamiltonian of the mentioned system will coincide with that of widely used simple models (see e.g. [16, 17, 18]) of quasi-free (conduction) electrons in movable (vibrating) crystal lattice:

$$\begin{aligned} H &= H_e + H_{ph} + H_{int} , \\ H_e &= \frac{p^2}{2m} , \quad H_{ph} = \sum_k \hbar \omega_k a_k^\dagger a_k , \\ H_{int} &= \frac{1}{\sqrt{\Omega}} \sum_k [c_k^* e^{ikr} a_k + c_k e^{-ikr} a_k^\dagger] \end{aligned} \quad (1)$$

Here Ω is the system's volume, so that wave vectors in the sums are separated by reciprocal volume $d^3k = (2\pi)^3/\Omega$, and

$$c_{-k} = c_k^* \quad (2)$$

for any wave vector k .

Being interested first of all in statistics of the electron's, or BP's, walk, we must consider the von Neumann evolution equation,

$$i\hbar \dot{\rho} = [H, \rho] \equiv H\rho - \rho H , \quad (3)$$

for full density matrix of the system, ρ . If at that we are interested first of all in finite temperatures, $T \equiv \beta^{-1} > 0$, then it is reasonable to describe phonon dependence of the ρ in terms of coherent states and the remarkable “Sudarshan-Glauber P-representation” [19, 20, 21] :

$$\rho = \int |\alpha\rangle\langle\alpha| P(\alpha, \alpha^*) d\alpha^* d\alpha , \quad (4)$$

where $\alpha = \{\alpha_k\}$, $|\alpha\rangle\langle\alpha| = \prod_k |\alpha_k\rangle\langle\alpha_k|$, $d\alpha^* d\alpha = \prod_k d\alpha_k^* d\alpha_k = \prod_k d\Re \alpha_k d\Im \alpha_k$, and the “quasi-probability density” [19, 20, 21], $P(\alpha, \alpha^*)$, in respect to the electron’s variables still acts as an operator. Taking into account that (for any k)

$$\begin{aligned} a |\alpha\rangle\langle\alpha| &= \alpha |\alpha\rangle\langle\alpha| , \quad a^\dagger |\alpha\rangle\langle\alpha| = (\partial + \alpha^*) |\alpha\rangle\langle\alpha| , \\ |\alpha\rangle\langle\alpha| a &= (\partial^* + \alpha) |\alpha\rangle\langle\alpha| , \quad |\alpha\rangle\langle\alpha| a^\dagger = \alpha^* |\alpha\rangle\langle\alpha| , \end{aligned} \quad (5)$$

with

$$\partial \equiv \frac{\partial}{\partial \alpha} , \quad \partial^* \equiv \frac{\partial}{\partial \alpha^*} ,$$

one can transform Eq.3 into

$$\begin{aligned} i\hbar \dot{P} &= [H_e, P] + \sum_k \hbar\omega_k (\alpha_k^* \partial_k^* - \alpha_k \partial_k) P + \\ &+ \frac{1}{\sqrt{\Omega}} \sum_k \{ c_k^* \alpha_k [e^{ikr}, P] + c_k \alpha_k^* [e^{-ikr}, P] + c_k^* \partial_k^* P e^{ikr} - c_k e^{-ikr} \partial_k P \} \end{aligned} \quad (6)$$

Further, it will be convenient to treat an electron dependence of P in the coordinate representation, introducing function $\langle r|P|r'\rangle$ instead of the operator P , with $|r\rangle$ denoting eigenstates of the electron’s coordinate operator r . Then it is convenient also to introduce new independent variables:

$$\begin{aligned} X &= \frac{r + r'}{2} , \quad Y = r - r' , \\ A_k &= \alpha_k e^{ikX} , \quad A_k^* = \alpha_k^* e^{-ikX} \end{aligned} \quad (7)$$

Designating the matrix element $\langle r|P|r'\rangle$ by the same symbol P ,

$$\langle r|P|r'\rangle \equiv P(t, X, Y, A, A^*) ,$$

we obtain for it from Eq.6 the equivalent equation

$$\begin{aligned} i\hbar \dot{P} &= -\frac{\hbar^2}{m} \nabla_Y \left[\nabla_X + \sum_k ik (A_k \partial_k - A_k^* \partial_k^*) \right] P + \\ &+ \sum_k \hbar\omega_k (A_k^* \partial_k^* - A_k \partial_k) P + \\ &+ \frac{1}{\sqrt{\Omega}} \sum_k [(e^{ikY/2} - e^{-ikY/2}) (c_k^* A_k - c_k A_k^*) + e^{-ikY/2} (c_k^* \partial_k^* - c_k \partial_k)] P \end{aligned} \quad (8)$$

with

$$\nabla_X = \frac{\partial}{\partial X} , \quad \nabla_Y = \frac{\partial}{\partial Y} , \quad \partial_k = \frac{\partial}{\partial A_k} , \quad \partial_k^* = \frac{\partial}{\partial A_k^*}$$

The Fourier transformation $\int \exp(-ipY/\hbar) P d^3Y / (2\pi\hbar)^3$ gives the Wigner representation with P becoming literally “quasi-probability density”.

2.2. Initial conditions

Being interested first of all in thermodynamically equilibrium electron's "Brownian motion", it is natural to start from such initial density matrix, - e.g. at $t = 0$, - which says that the phonon subsystem has canonical equilibrium probability distribution with given temperature $T = 1/\beta$ while the electron is located in vicinity of some given point of space, - e.g. coordinate origin, - and has equilibrium Maxwellian momentum distribution corresponding to the same temperature. Such the density matrix can be expressed by formulas

$$\begin{aligned} \rho(t=0) &\propto |\Psi\rangle\langle\Psi| \exp(-\beta H_{ph}) , \\ \Psi(r) &= \left(\frac{2mT}{\pi\hbar^2}\right)^{3/4} \exp\left(-\frac{mT}{\hbar^2} r^2\right) \end{aligned} \quad (9)$$

In order to rewrite this in the P-representation, notice [21] that for any particular phonon mode

$$(1 - e^{-\beta\hbar\omega}) \exp(-\beta\hbar\omega a^\dagger a) = \int |\alpha\rangle\langle\alpha| f(\omega, \alpha, \alpha^*) d\alpha^* d\alpha ,$$

where

$$\begin{aligned} f(\omega, \alpha, \alpha^*) &\equiv \frac{1}{\pi N(\omega)} \exp\left[-\frac{|\alpha|^2}{N(\omega)}\right] , \\ N(\omega) &\equiv \frac{1}{\exp(\beta\hbar\omega) - 1} \end{aligned} \quad (10)$$

Therefore initial condition (9) is equivalent to

$$P(t=0) = \Psi(r)\Psi^*(r') \prod_k f(\omega_k, \alpha_k, \alpha_k^*) , \quad (11)$$

which is normalized distribution.

Obviously, such initial condition as (9) and (11) has serious defect: it neglects statistical correlations between electron and phonons inevitably arising from their interactions. However, this defect may be simultaneously its advantage: it helps to trace the development of the correlations during subsequent evolution directed by Eqs.3,6 or 8. Therefore, it is reasonable to accept such initial condition at least for a time.

2.3. Marginal distributions

The object of our direct interest is the marginal density matrix of electron,

$$\rho_0 = \prod_k \text{Tr}_k \rho , \quad (12)$$

- with Tr_k being trace over states of phonon mode k , - or corresponding marginal distribution

$$P_0 = \int P \prod_k d\alpha_k^* d\alpha_k = \int P \prod_k dA_k^* dA_k$$

Alternative description of its evolution is presented by hierarchy of equations for the chain of various marginal, or particular, distributions

$$P_n = \int P \prod_{k \neq K_n} dA_k^* dA_k ,$$

where $K_n = \{k_1 \dots k_n\}$ is a set of wave vectors of n **different** phonon modes. Applying these integrations to Eqs.6 or 8, we come to hierarchy

$$\begin{aligned} \dot{P}_n = & -\hat{V} \nabla_X P_n + \sum_{k \in K_n} [\hat{L}_k + \mu \hat{A}_k + \mu \hat{D}_k] P_n + \\ & + \mu \sum_{q \neq K_n} \int \hat{A}_q P_{n+1} dA_q^* dA_q , \end{aligned} \quad (13)$$

where P_{n+1} corresponds to $K_{n+1} = \{K_n, q\} = \{k_1 \dots k_n, q\}$, and we introduced operators

$$\begin{aligned} \hat{V} & \equiv -\frac{i\hbar}{m} \nabla_Y , \\ \hat{L}_k & \equiv i(\omega_k - k\hat{V})(A_k \partial_k - A_k^* \partial_k^*) , \\ \hat{A}_k & \equiv (i\hbar\sqrt{\Omega_0})^{-1} (e^{ikY/2} - e^{-ikY/2})(c_k^* A_k - c_k A_k^*) , \\ \hat{D}_k & \equiv (i\hbar\sqrt{\Omega_0})^{-1} e^{-ikY/2} (c_k^* \partial_k^* - c_k \partial_k) , \end{aligned} \quad (14)$$

$$(15)$$

and besides the small parameter

$$\mu \equiv \sqrt{\Omega_0/\Omega}$$

with Ω_0 being some fixed “microscopic” volume.

It will be useful to treat these equations in terms of new functions Q_n defined by

$$P_n = Q_n \prod_{k \in K_n} f_k ,$$

with shortened notations

$$\begin{aligned} f_k & \equiv f(\omega_k, A_k, A_k^*) = (\pi N_k)^{-1} \exp(-|A_k|^2/N_k) , \\ N_k & \equiv N(\omega_k) = 1/[\exp(\beta\hbar\omega_k) - 1] \end{aligned}$$

They undergo hierarchy of equations

$$\begin{aligned} \dot{Q}_n = & -\hat{V} \nabla_X Q_n + \sum_{k \in K_n} [\hat{L}_k + \mu \hat{A}_k + \mu f_k^{-1} \hat{D}_k f_k] Q_n + \\ & + \mu \sum_{q \neq K_n} \int f_q \hat{A}_q Q_{n+1} dA_q^* dA_q = \\ = & -\hat{V} \nabla_X Q_n + \sum_{k \in K_n} [\hat{L}_k + \mu \hat{B}_k + \mu \hat{D}_k] Q_n + \\ & + \mu \sum_{q \neq K_n} \int f_q \hat{A}_q Q_{n+1} dA_q^* dA_q , \end{aligned} \quad (16)$$

where

$$\hat{B}_k \equiv \hat{A}_k + (i\hbar\sqrt{\Omega_0})^{-1} e^{-ikY/2} (c_k A_k^* - c_k^* A_k)/N_k \quad (17)$$

Clearly, $Q_0 \equiv P_0$, and initial conditions to Eqs.16 following from (11) are

$$Q_n(t=0) = \Psi(r)\Psi^*(r') \quad (18)$$

Equations (13) and (16) are analogues of the BBGKY-like hierarchies of equations for BP in a classical ideal gas of atoms [1, 2, 3, 4, 5] or generally in a simple fluid [2, 4, 7, 8, 9, 12], with the last terms in (13) and (16) playing role of “collision integrals”.

The problem is again in investigation of properties of the BP’s (electron’s) distribution function $P_0 = P_0(t, X, Y)$ what follow from Eqs.13 or 16.

3. Thermodynamic limit and shortened hierarchy of evolution equations

In order to grant to the BP (electron) large enough space for random walk and make the phonon subsystem to be as much as possible powerful thermostat, we must go to the thermodynamic limit $\Omega \rightarrow \infty$, $\mu \rightarrow 0$.

3.1. Preliminary discussion

Notice that under this limit, firstly, perturbation of any particular phonon mode by its interaction with electron is vanishingly small, so that

$$P_n \rightarrow P_0 \prod_{k \in K_n} f_k, \quad Q_n \rightarrow Q_0 = P_0 \quad (19)$$

for all n . More precisely, the Eqs.8,13 and 16, along with initial conditions (11) and (18), do imply that

$$P = \left\{ \sum_{s=0}^{\infty} \mu^s P^{(s)} \right\} \prod_k f_k,$$

where $P^{(s)}$ is a s -order polynomial of phonon variables A_k^*, A_k , with coefficients being some functions of t, X and Y . Correspondingly,

$$Q_n = \sum_{s=0}^{\infty} \mu^s P_n^{(s)}, \quad (20)$$

where $P_n^{(s)}$ is a s -order polynomial of A_k^*, A_k with $k \in K_n$ and with coefficients representing some functions of t, X, Y and μ .

At the same time, secondly, because of finiteness of limits

$$\mu^2 \sum_q \dots \rightarrow \Omega_0 \int \dots d^3q / (2\pi)^3,$$

- with dots replacing a “good” function of q (which is the case under natural restrictions on $|c_k|^2$), - summary perturbation of electron by all the phonon modes acquires at $\mu \rightarrow 0$ a constant value independent on μ and indifferent to exclusion (or neglect) of any finite set of phonon modes. Therefore, coefficients of all the polynomials $P_n^{(s)}$ in fact become independent on μ .

These remarks show that, in agreement with (19), all $P_n^{(0)}$ with $n > 0$ give no essential information in addition to one already contained in P_0 . Actually, at $n = 1$ a portion of new additional information is presented by second term of expansion (20), i.e. by coefficients of the first-order polynomial $P_1^{(1)}$. Indeed, according to the first of Eqs.13 and 16 (for P_0 and Q_0), at $\mu \rightarrow 0$ just $P_1^{(1)}$ directly and completely determines “collision integrals” in these equations (since $\mu^2 \sum_q \dots \rightarrow \Omega_0 \int \dots d^3q / (2\pi)^3$ and, clearly, contributions from all $P_1^{(s)}$ with $s > 1$ disappear in the thermodynamic limit).

Similarly, any of $P_n^{(1)}$ with $n > 1$ gives no new information in comparison with that already contained in coefficients of $P_1^{(1)}$. Considering evolution of these coefficients, - with the help of second terms of (13) or (16) (at $n = 1$), - one can see that at $\mu \rightarrow 0$ they are exactly determined, through corresponding components of collision integrals, by coefficients of polynomial $P_2^{(2)}$. Or, to be more precise, four coefficient of bilinear part of $P_2^{(2)}$.

Continuation of such reasonings leads to conclusion that hierarchy of functions, which unambiguously determine evolution of the electron’s distribution function P_0 , in the thermodynamic limit reduces to coefficients of **purely multi-linear parts** of polynomials $P_n^{(n)}$ (which is easy understandable: since density of phonon modes in k -space tend to infinity, probability of strong excitation of one and the same mode becomes infinitely small as compared with probability of weak excitation of many different, let close in k -space, modes).

In order to confirm these statements, we have to extract from (13) or (16) a closed hierarchy of shortened equations for the mentioned coefficients.

3.2. Derivation of shortened equations

First, let us define them formally:

$$\Delta_n \equiv \lim_{\mu \rightarrow 0} \frac{1}{\mu^n} \left[\left(\prod_{k \in K_n} \partial_k^{\sigma_k} \right) Q_n \right]_{A=A^*=0} \equiv \hat{\Delta}_n Q_n , \quad (21)$$

where σ_k can take one of two symbolic values, “+” or “-”, and we introduced new designations (more comfortable for further manipulations):

$$A_k^- \equiv A_k , \quad A_k^+ \equiv A_k^* , \quad \partial_k^- \equiv \partial_k , \quad \partial_k^+ \equiv \partial_k^*$$

Applying such operation to $(n+1)$ -th of Eqs.16 (for Q_n), consider different terms there separately.

For the second right-hand term one readily obtains

$$\hat{\Delta}_n \hat{L}_k Q_n = -i\sigma_k (\omega_k - k\hat{V}) \Delta_n , \quad (22)$$

where k is one of elements of the set K_n from (21).

The third term needs in more careful consideration. Application of $\hat{\Delta}_n$ to it gives

$$\begin{aligned} \hat{\Delta}_n \mu \hat{B}_k Q_n &= -(i\hbar\sqrt{\Omega_0})^{-1} \sigma_k c_k^{\sigma_k} \times \\ &\times [e^{ikY/2} - e^{-ikY/2}(1 + N_k^{-1})] \hat{\Delta}_{n-1} Q_n |_{A_k=A_k^*=0} , \end{aligned} \quad (23)$$

where

$$c_k^- \equiv c_k^* , \quad c_k^+ \equiv c_k ,$$

and, clearly, $\hat{\Delta}_{n-1}$ corresponds to the set $K_{n-1} = K_n \ominus k$.

Next, to express $\hat{\Delta}_{n-1} Q_n$ in (23) through functions (21), we should take into account, firstly, that by the Q_n -s definition

$$\int Q_n f_q dA_q^* dA_q = Q_{n-1} \quad (K_n = K_{n-1} \oplus q) \quad (24)$$

Therefore

$$\int \left[\frac{1}{\mu^{n-1}} \prod_{k \in K_{n-1}} \partial_k^{\sigma_k} Q_n \right] f_q dA_q^* dA_q = \frac{1}{\mu^{n-1}} \prod_{k \in K_{n-1}} \partial_k^{\sigma_k} Q_{n-1} \quad (25)$$

Secondly, turning here all A_k^* , A_k with $k \in K_{n-1}$ into zeros, in view of the expansion (20) for the square bracket on the left we can write

$$[\dots]_{A_{K_{n-1}}=A_{K_{n-1}}^*=0} = [\dots]_{A=A^*=0} + \sum_{s=1}^{\infty} \mu^s p^{(s)}(A_q, A_q^*) , \quad (26)$$

where $A_{K_{n-1}}$ means any of A_k with $k \in K_{n-1}$, while A any of all phonon variables with wave vectors from K_n , and $p^{(s)}(A_q, A_q^*)$ is s -order homogeneous polynomial resulting from $P_n^{(n-1+s)}$.

Combining (25) and (26), in the limit $\mu \rightarrow 0$ we come to equality

$$\hat{\Delta}_{n-1} Q_n |_{A_q=A_q^*=0} = \Delta_{n-1} \quad (K_n = K_{n-1} \oplus q) \quad (27)$$

With its help Eq.23 yields

$$\hat{\Delta}_n \mu \hat{B}_k Q_n = -(i\hbar\sqrt{\Omega_0})^{-1} \sigma_k c_k^{\sigma_k} [e^{ikY/2} - e^{-ikY/2}(1 + N_k^{-1})] \Delta_{n-1} \quad (28)$$

(with $K_{n-1} = K_n \ominus k$).

Now, consider action of $\hat{\Delta}_n$ onto fourth right-hand term of (16):

$$\begin{aligned} \hat{\Delta}_n \mu \hat{D}_k Q_n &= \lim_{\mu \rightarrow 0} \mu^2 (i\hbar\sqrt{\Omega_0})^{-1} e^{-ikY/2} \times \\ &\times \left[\frac{1}{\mu^{n+1}} (c_k^* \partial_k^* - c_k \partial_k) \prod_{q \in K_n} \partial_q^{\sigma_q} Q_n \right]_{A=A^*=0} \end{aligned}$$

Evidently, the square bracket here has a finite limit at $\mu \rightarrow 0$, again thanks to (20). Therefore the total expression tends to zero, so that

$$\hat{\Delta}_n \mu \hat{D}_k Q_n = 0 \quad (29)$$

At last, consider action of $\hat{\Delta}_n$ onto “collision integrals in (16). At that, we should use identities

$$\begin{aligned} \int f_q A_q [\dots] dA_q^* dA_q &= \int \{-N_q \partial_q^* f_q\} A_q [\dots] dA_q^* dA_q = \\ &= N_q \int f_q \partial_q^* [\dots] dA_q^* dA_q , \\ \int f_q A_q^* [\dots] dA_q^* dA_q &= N_q \int f_q \partial_q [\dots] dA_q^* dA_q , \end{aligned} \quad (30)$$

where square bracket means arbitrary function of the phonon variables. Hence, collision integrals can be rewritten as follows,

$$\begin{aligned}
\widehat{C}_{n+1} Q_{n+1} &\equiv \mu \sum_{q \neq K_n} \int f_q \widehat{A}_q Q_{n+1} dA_q^* dA_q = \\
&= \mu (i\hbar\sqrt{\Omega_0})^{-1} \sum_{q \neq K_n} (e^{iqY/2} - e^{-iqY/2}) N_q \times \\
&\times \int f_q (c_q^* \partial_q^* - c_q \partial_q) Q_{n+1} dA_q^* dA_q,
\end{aligned} \tag{31}$$

and

$$\begin{aligned}
\widehat{\Delta}_n \widehat{C}_{n+1} Q_{n+1} &= \\
&= (i\hbar\sqrt{\Omega_0})^{-1} \lim_{\mu \rightarrow 0} \mu^2 \sum_{q \neq K_n} (e^{iqY/2} - e^{-iqY/2}) N_q \times \\
&\times \int f_q \sum_{\sigma_q \in \{+, -\}} \sigma_q c_q^{-\sigma_q} \left[\frac{1}{\mu^{n+1}} \partial_q^{\sigma_q} \prod_{k \in K_n} \partial_k^{\sigma_k} Q_{n+1} \right]_{A_{K_n}=A_{K_n}^*=0} dA_q^* dA_q
\end{aligned}$$

Here A_{K_n} means any of A_k with $k \in K_n$. According to (20), the square bracket here behaves at $\mu \rightarrow 0$ in full analogy with that in (26),

$$[\dots]_{A_{K_n}=A_{K_n}^*=0} = [\dots]_{A=A^*=0} + \sum_{s=1}^{\infty} \mu^s \widetilde{p}^{(s)}(A_q, A_q^*),$$

with polynomials $\widetilde{p}^{(s)}(A_q, A_q^*)$ now produced from $P_{n+1}^{(n+1+s)}$.

Consequently, after performing the limit and transforming the sum over q into integral, we obtain

$$\begin{aligned}
\widehat{\Delta}_n \widehat{C}_{n+1} Q_{n+1} &= \\
&= \frac{\sqrt{\Omega_0}}{i\hbar} \sum_{\sigma \in \{+, -\}} \sigma \int c_q^{-\sigma} (e^{iqY/2} - e^{-iqY/2}) N_q \Delta_{n+1} \frac{d^3 q}{(2\pi)^3},
\end{aligned} \tag{32}$$

where $K_{n+1} = K_n \oplus q$ and σ corresponds to q (so that one can replace dummy variable q by k_{n+1} and its pair σ by σ_{n+1}).

As the result of relations (22), (28), (29) and (32), we come under the thermodynamic limit to the following exact short-cut (nevertheless, infinite) hierarchy of evolution equations:

$$\begin{aligned}
\dot{\Delta}_n &= -\widehat{V} \nabla_X \Delta_n - i \sum_{k \in K_n} \sigma_k (\omega_k - k\widehat{V}) \Delta_n - \\
&- (i\hbar\sqrt{\Omega_0})^{-1} \sum_{k \in K_n} \sigma_k c_k^{\sigma_k} [e^{ikY/2} - e^{-ikY/2} (1 + N_k^{-1})] \Delta_{n-1} + \\
&+ \frac{\sqrt{\Omega_0}}{i\hbar} \sum_{\sigma \in \{+, -\}} \sigma \int c_q^{-\sigma} (e^{iqY/2} - e^{-iqY/2}) N_q \Delta_{n+1} \frac{d^3 q}{(2\pi)^3}
\end{aligned} \tag{33}$$

Here $K_{n-1} = K_n \ominus k$ and $\Sigma_{n-1} = \Sigma_n \ominus \sigma_k$ in the third term on the right and $K_{n+1} = K_n \oplus q$ and $\Sigma_{n+1} = \Sigma_n \oplus \sigma$ in the last term (collision integral), with Σ_n denoting sets of the two-fold indices paired with wave vectors.

Recall that all wave vectors in any of the sets K_n have appeared different one from another, so that “collision integral” in the last term, strictly speaking, does not include (infinitely small neighborhoods of) n points $q \in K_n$. However, both the Δ_n ’s definition (21) and structure of Eqs.33 allow to extend functions Δ_n to coinciding wave vectors by continuity.

Initial conditions to Eqs.33, what literally follow from (9) or (18),

$$\Delta_n(t=0) = \delta_{n0} \Psi(r) \Psi^*(r') , \quad (34)$$

show that there is no need in any a priori assumptions about these functions.

As for the electron’s probability distribution function $\Delta_0 = P_0$, in essence, Eqs.33 altogether form an exact time-non-local kinetic equation for it. At that, its part of the initial conditions can be chosen arbitrarily.

3.3. Stationary solution, equilibrium distribution, and thermodynamically improved initial conditions

The Eqs.33 by their derivation describe an electron in infinitely large phonon (or generally boson) thermostat formed by continuum of modes each being initially in canonical equilibrium state. Therefore, firstly, stationary solution of Eqs.33, - to be denoted as Δ_n^{eq} , - must represent thermodynamically equilibrium distribution of the electron’s momentum under fully uncertain electron’s position, so that

$$\nabla_X \Delta_n^{eq} = \dot{\Delta}_n^{eq} = 0$$

Such solution can not be strictly normalized to unit, - in respect to both momentum and coordinate of the electron, - but it can be normalized in respect to momentum only:

$$\Delta_0^{eq}(Y=0) = 1$$

Secondly, time-dependent (strictly normalized) solution of Eqs.33 must tend to such stationary equilibrium solution in the sense that

$$\lim_{t \rightarrow \infty} \int \Delta_n d^3X = \Delta_n^{eq}$$

The functions Δ_n^{eq} contain complete information about equilibrium statistical correlations between electron’s momentum and thermostat. Hence, if we want to take into account these inevitable correlations from the very beginning, we should replace initial conditions (34) by

$$\Delta_n(t=0) = W_0(X) \Delta_n^{eq} , \quad (35)$$

where a normalized probability density $W_0(X)$ introduces initial localization of the electron in configurational space. Then time-dependent solution of Eqs.33 will highlight specific “historical” statistical correlations [2, 4] arising between current state of the thermostat and summary path of BP (electron) during all the time interval $(0, t)$. It may be said that the “equilibrium correlations” describe momentary electron-phonons interaction while the “historical” ones describe a heritage of past interactions.

4. Equivalent representations and generating functional of electron-phonon correlations

A suitable change of the variables Δ_n at $n > 0$ helps to remove the formally arbitrary parameter Ω_0 and fashion Eqs.33 to a more visual form. For instance, introducing, instead of Δ , functions D_n defined by

$$\Delta_n = D_n \prod_{k \in K_n} s_{k\sigma_k} \ , \quad s_{k\sigma} = -\frac{c_k^\sigma}{\hbar\omega_k N_k \sqrt{\Omega_0}} \quad (36)$$

at $n > 0$ and $D_0 = \Delta_0$, we transform Eqs.33 into

$$\begin{aligned} \dot{D}_n = & -\hat{V} \nabla_X D_n - i \sum_{k \in K_n} \sigma_k (\omega_k - k\hat{V}) D_n - \\ & - i \sum_{k \in K_n} \sigma_k \omega_k [e^{ikY/2} N_k - e^{-ikY/2} (N_k + 1)] D_{n-1} + \\ & + i \sum_{\sigma \in \{+, -\}} \sigma \int \frac{|c_q|^2}{\hbar^2 \omega_k} (e^{iqY/2} - e^{-iqY/2}) D_{n+1} \frac{d^3 q}{(2\pi)^3} \ , \end{aligned} \quad (37)$$

with the same meaning of σ and q as before.

4.1. Kinetic equation and infinite hierarchy of irreducible correlations

Physical and statistical interpretation of just made change becomes clear if notice, firstly, that, - as one can prove in the spirit of above consideration, - at different wave vectors definition (21) is equivalent to

$$\Delta_n = \prod_{k \in K_n} \frac{1}{N_k} \lim_{\mu \rightarrow 0} \frac{1}{\mu^n} \int P_n \prod_{k \in K_n} A_k^{-\sigma_k} dA_k^* dA_k \quad (38)$$

Secodly, if the electron was immovable (“infinitely hard”) then perturbation of all the phonon amplitudes would reduce to their constant shifts by

$$\overline{A}_k^\sigma = -\frac{c_k^\sigma}{\hbar\omega_k \sqrt{\Omega}} = \mu s_{k\sigma} N_k$$

Comparison of this expression with (36) shows that

$$D_n = \int P_n \prod_{k \in K_n} \frac{A_k^{-\sigma_k}}{\overline{A}_k^{\sigma_k}} dA_k^* dA_k \ ,$$

that is D_n ($n > 0$) represent joint perturbation of n (different) phonon modes measured in natural units of their separate static perturbations. Under the Wigner representation of D_n ’s electron dependence this relation can be rewritten as

$$D_n(t, X, p, K_n, \Sigma_n) = D_0(t, X, p) \left\langle \prod_{k \in K_n} \frac{A_k^{-\sigma_k}(t)}{\overline{A}_k^{\sigma_k}} \right\rangle_{X, p} \ , \quad (39)$$

where the angle brackets mean conditional average of (quasi-classic) phonon amplitudes at given electron’s position and momentum.

At that, our basic definition (21) prompts that functions D_n ($n > 0$) indeed must be extended to (zero-measure subspaces of) coinciding wave vectors by continuity.

From the Hamiltonian (1) it is seen that, in case of immovable (“static”) electron all phonon modes would be statistically independent one on another, and all the statistical moments in (39) factored into products of first-order moments (average values). In reality, however, electron’s motion couples different phonon modes one with another making them statistically dependent. Therefore their statistical moments consist of not only average values but also second- and higher-order irreducible correlations (cumulants). They reflect both current and old interactions (figuratively speaking, “Correlations = Interactions + History”). The Eqs.37 exactly describe influence of “current” onto dynamics and “old” onto statistics of electron motion (for detail explanations of nature of “historical” correlations see e.g. [7, 10, 11, 12, 13, 14]).

4.2. Generating functional

It may be convenient to accumulate all the components of Eqs.37 into a single object, such as vector $\{D_0, D_1, \dots\}$ in the corresponding Fock space or the generating functional:

$$\mathcal{D} \equiv D_0 + \sum_{n=1}^{\infty} \frac{1}{n!} \sum_{\sigma_1 \dots \sigma_n} \int D_n \prod_{j=1}^n z_{\sigma_j}(k_j) d^3 k_j$$

so that

$$D_n = \left[\prod_{j=1}^n \frac{\delta}{\delta z_{\sigma_j}(k_j)} \mathcal{D} \right]_{z=0}$$

In terms of this functional the whole infinite hierarchy (37) reduces to single equation

$$\begin{aligned} \dot{\mathcal{D}} &= -\hat{V} \nabla_X \mathcal{D} + \sum_{\sigma} \int d^3 k \, z_{\sigma}(k) \hat{L}_{k\sigma} \frac{\delta}{\delta z_{\sigma}(k)} \mathcal{D} + \\ &+ \sum_{\sigma} \int d^3 k \, z_{\sigma}(k) \hat{B}_{k\sigma} \mathcal{D} + \sum_{\sigma} \int d^3 k \, \hat{A}_{k\sigma} \frac{\delta}{\delta z_{\sigma}(k)} \mathcal{D} \equiv \\ &\equiv -\hat{V} \nabla_X \mathcal{D} + \hat{\mathcal{L}} \left\{ z, \frac{\delta}{\delta z} \right\} \mathcal{D} , \end{aligned} \quad (40)$$

where

$$\begin{aligned} \hat{L}_{k\sigma} &\equiv -i\sigma (\omega_k - k\hat{V}) , \\ \hat{B}_{k\sigma} &\equiv -i\sigma \omega_k [e^{ikY/2} N_k - e^{-ikY/2} (N_k + 1)] , \\ \hat{A}_{k\sigma} &\equiv i\sigma \frac{|c_k|^2}{\hbar^2 \omega_k (2\pi)^3} (e^{ikY/2} - e^{-ikY/2}) , \end{aligned} \quad (41)$$

and \hat{V} is electron’s velocity operator defined in (14).

Quite similarly we can introduce generating functional Δ for the correlation functions Δ_n and then write out similar evolution equation for it. Clearly, both they are

connected with \mathcal{D} and Eq.40 through simple scale transformation of the test function $z = z_\sigma(k)$,

$$\begin{aligned}\Delta\{z\} &= \mathcal{D}\{sz\} , \\ \dot{\Delta} &= -\widehat{V}\nabla_X \Delta + \widehat{\mathcal{L}}\left\{sz, \frac{\delta}{s\delta z}\right\} \Delta ,\end{aligned}\quad (42)$$

with above introduced multiplier $s = s_{k\sigma}$.

The functional evolution equation (40) is direct formal analogue of equations for BP in fluids investigated in [1, 2, 3, 4, 5]. Investigation of Eq.40 or Eq.42 may be subject of future works. At the rest of this one we confine ourselves by discussion of some consequences and modifications of the Eqs.40 and 42.

5. Generalizations and special cases

5.1. Other particles and external potentials

In presence of additional “external” particles, interacting with the phonon thermostat and distributed with given density $\nu(r)$, the Hamiltonian (1) transforms as

$$H \Rightarrow H + H_a , \quad H_a = \frac{1}{\sqrt{\Omega}} \sum_k [c_k^* \nu_k^* a_k + c_k \nu_k a_k^\dagger] , \quad (43)$$

where

$$\nu_k \equiv \int e^{-ikr} \nu(r) d^3r$$

Then after repeating, for new Hamiltonian (43), the above expounded derivation of shortened equations, one easy comes to Eq.40 with modified third right-side term:

$$\widehat{\mathcal{L}} \Rightarrow \widehat{\mathcal{L}}' \equiv \widehat{\mathcal{L}} + \sum_\sigma \int d^3k z_\sigma(k) [i\sigma \omega_k \nu_k^\sigma \exp(i\sigma kX)] , \quad (44)$$

with $\nu_k^+ \equiv \nu_k$, $\nu_k^- \equiv \nu_k^*$. This addition to evolution operator $\widehat{\mathcal{L}}$ does not include the electron momentum related operators \widehat{V} and Y . Therefore it does not create additional electron-phonon correlations (instead producing momentum-independent shifts of mean values $\langle A_k^\sigma \rangle$ only). Hence, it can be removed by representing solution \mathcal{D}' to the modified evolution equation,

$$\dot{\mathcal{D}}' = -\widehat{V}\nabla_X \mathcal{D}' + \widehat{\mathcal{L}}' \left\{z, \frac{\delta}{\delta z}\right\} \mathcal{D}' , \quad (45)$$

in the form

$$\mathcal{D}' = \mathcal{D} \exp \left[\sum_\sigma \int z_\sigma(k) \nu_k^\sigma e^{i\sigma kX} d^3k \right] \quad (46)$$

Then a simple algebra turns the Eq.45 into

$$\dot{\mathcal{D}} = -\widehat{V}\nabla_X \mathcal{D} + \frac{U(r) - U(r')}{i\hbar} \mathcal{D} + \widehat{\mathcal{L}} \left\{z, \frac{\delta}{\delta z}\right\} \mathcal{D} , \quad (47)$$

where $r, r' = X \pm Y/2$ and

$$U(r) = - \int \frac{|c_k|^2}{\hbar\omega_k} [\nu_k e^{ikr} + \nu_k^* e^{-ikr}] \frac{d^3k}{(2\pi)^3} \quad (48)$$

is potential of effective (mediated by phonons) interaction of our electron with the “external” particles.

This result is not significant in itself, for it follows already directly from the Hamiltonian (43), with the help of boson operator amplitudes shifting

$$A_k + \frac{c_k \nu_k}{\hbar\omega_k \sqrt{\Omega}} \Rightarrow A_k, \quad A_k^* + \frac{c_k^* \nu_k^*}{\hbar\omega_k \sqrt{\Omega}} \Rightarrow A_k^*$$

Nevertheless, it is important for us since it verifies faultlessness of our derivation of Eqs.33 or Eqs.37 and thus Eqs.40.

5.2. Many phonon branches (boson thermostats)

One of generalizations of the model (1), which may be necessary in its applications, is accounting for presence of several types, or “branches”, of phonon modes, with different dispersion laws and/or couplings to the electron. Obviously, to perform such a generalization, it is sufficient to impart an additional index, - let “b”, - to the phonon amplitudes A_k, A_k^* , frequencies ω_k and coupling constants c_k^σ , the sets of n such indices to the correlation functions Δ_n and D_n and, correspondingly, the same additional index to the test function $z_\sigma(k)$ and all operators (41), so that

$$\sum_\sigma \int d^3k \dots \Rightarrow \sum_b \sum_\sigma \int d^3k \dots$$

in Eqs.40 and other functional evolution equations.

Equivalently, we can divide the corresponding total functional evolution operator, $\hat{\mathcal{L}}$, into sum of operators $\hat{\mathcal{L}}^{(b)}$ each relating to one of the phonon branches, so that the Eq.40 takes form

$$\dot{\mathcal{D}} = -\hat{V} \nabla_X \mathcal{D} + \sum_b \hat{\mathcal{L}}^{(b)} \left\{ z_b, \frac{\delta}{\delta z_b} \right\} \mathcal{D} \quad (49)$$

Advantage of this representation is in that it naturally extends to the case of different temperatures of various phonon branches (in other words, boson thermostats), which is obviously allowed by our above analysis.

5.3. Application to electron in static disorder

Let us consider a quantum particle (“electron”) in Gaussian static random potential $\Phi(r)$ with zero mean value and correlation function

$$\langle \Phi(r_1) \Phi(r_2) \rangle = \int |c_k|^2 e^{ik(r_1-r_2)} d^3k / (2\pi)^3 \quad (50)$$

We assume that the electron’s density matrix, ρ_e , at some time moment, $t = 0$, is known, $\rho_e(t = 0) = \rho_{e0}$, and want to know its mean value at $t \neq 0$,

$$\bar{\rho}_e(t) = \langle e^{-iHt/\hbar} \rho_{e0} e^{iHt/\hbar} \rangle, \quad (51)$$

where $H = p^2/2m + \Phi(r)$ is Hamiltonian of the system.

In calculations of this quantity the so-called operator representation of random processes and fields may be useful [22]. For instance, any statistical moment of Gaussian field with auto-correlation (50) can be represented as

$$\langle \prod_j \Phi(r_j) \rangle = \langle 0 | \prod_j \hat{\Phi}(r_j) | 0 \rangle ,$$

where

$$\hat{\Phi}(r) = \frac{1}{\sqrt{\Omega}} \sum_k [c_k^* e^{ikr} a_k + c_k e^{-ikr} a_k^\dagger]$$

is quantum boson field operator with coefficients satisfying (2), $\Omega \rightarrow \infty$, and $|0\rangle$ is the “vacuum state”. Therefore instead of (51) one can write

$$\bar{\rho}_e(t) = \langle 0 | e^{-iHt/\hbar} \rho_{e0} e^{iHt/\hbar} | 0 \rangle ,$$

now with $H = p^2/2m + \hat{\Phi}(r)$.

From here it follows that

$$\langle \Psi | \bar{\rho}_e(-t) | \Psi \rangle = \text{Tr}_e \rho_{e0} \{ \text{Tr}_b e^{-iHt/\hbar} | 0 \rangle \langle \Psi | \langle \Psi | e^{iHt/\hbar} \} , \quad (52)$$

where $|\Psi\rangle$ is arbitrary electron state and Tr_e , Tr_b mean traces over electron and boson degrees of freedom, respectively. Obviously, the expression in the braces is particular and peculiar case of the expression (12) arising from it under limits $T \rightarrow 0$ and $\omega_k \rightarrow 0$. At that, the first of these limits should be performed before the second, since transition to vacuum initial state of boson subsystem presumes that $N_k \rightarrow 0$.

To realize such ordered limits in Eqs.33, we firstly have to rescale functions Δ_n by

$$\Delta_n = D_n \prod_{k \in K_n} s_{k\sigma_k} , \quad s_{k\sigma} = -\frac{c_k^\sigma}{\hbar N_k \sqrt{\Omega_0}} ,$$

instead of (36), then set $N_k = 0$ and after that $\omega_k = 0$. This results in equations

$$\begin{aligned} \dot{D}_n(t, X, Y, K, \Sigma) &= [-\nabla_X + i \sum_{k \in K} \sigma_k k] \hat{V} D_n(t, X, Y, K, \Sigma) + \\ &+ i \sum_{k \in K} \sigma_k e^{-ikY/2} D_{n-1}(t, X, Y, K \ominus k, \Sigma \ominus \sigma_k) + \\ &+ i \sum_{\sigma} \sigma \int \frac{|c_q|^2}{\hbar^2} (e^{iqY/2} - e^{-iqY/2}) D_{n+1}(t, X, Y, K \oplus q, \Sigma \oplus \sigma) \frac{d^3 q}{(2\pi)^3} \end{aligned} \quad (53)$$

(with designations slightly different from that in Eqs.37 but clearly equivalent to them). According to (51) and (52), if $D_n(t)$ is solution of these equations under initial conditions

$$D_n(t=0) = \delta_{n0} |\Psi\rangle \langle \Psi| , \quad (54)$$

then

$$\langle \Psi | \bar{\rho}_e(-t) | \Psi \rangle = \text{Tr}_e \rho_{e0} D_0(t) \quad (55)$$

In particular, at $\rho_{e0} = |\Psi_0\rangle \langle \Psi_0|$ we obtain expression,

$$\langle |\langle \Psi | e^{iHt/\hbar} | \Psi_0 \rangle|^2 \rangle = \langle \Psi_0 | D_0(t) | \Psi_0 \rangle , \quad (56)$$

for mean probability of electron's transition between two given states Ψ_0 and Ψ .

Because of the “Anderson localization” of a part of electron states in random potential [23, 24, 25], the quantity (56) has non-zero limit at $t \rightarrow \pm\infty$ depending on both Ψ_0 and Ψ . This fact means that stationary solution of Eqs.53 is not unique but strongly degenerated (consequence of that now the “thermostat” has zero internal energy and thus no ability to “thermalize” electron). These multiple stationary solutions can be used to represent such (“eigen”) electron states which are correlated with the disorder (in contrast to a priory given states Ψ_0 and Ψ).

5.4. Naive “weak-coupling limit” and conventional kinetic equation

Returning to the case when $T \neq 0$ and $\omega_k \neq 0$, let us assume that some sufficient smallness of the coupling constants $|c_k|^2$, e.g. in the sense of

$$\gamma \equiv \int \frac{|c_k|^2}{\hbar^2 \omega_k} \frac{d^3 k}{(2\pi)^3} \ll \frac{T}{\hbar} ,$$

gives us rights to truncate the hierarchy (33) or, equivalently, (37) at its second level by setting $\Delta_2 = 0$ or $D_2 = 0$, i.e. neglecting third-order correlations (between electron and two phonon modes) and higher-order ones. Then, after transition to the Wigner representation, so that

$$\exp(\pm i k Y / 2) F(Y) \Rightarrow F(p \mp \hbar k / 2) ,$$

we come to equation

$$\begin{aligned} \dot{D}_0(t, X, p) = & -V \nabla_X D_0(t, X, p) + \sum_{\sigma} \int \frac{d^3 k}{(2\pi)^3} \frac{|c_k|^2}{\hbar^2} \int_0^t d\tau \times \\ & \times \left\{ \exp \left[-i\sigma \left(\omega_k - kV + \frac{\hbar k^2}{2m} \right) \tau \right] \times \right. \\ & \times [N_k D_0(t - \tau, X - \tau(V - \frac{\hbar k}{2m}), p - \hbar k) - \\ & - (N_k + 1) D_0(t - \tau, X - \tau(V - \frac{\hbar k}{2m}), p)] + \\ & + \exp \left[-i\sigma \left(\omega_k - kV - \frac{\hbar k^2}{2m} \right) \tau \right] \times \\ & \times [(N_k + 1) D_0(t - \tau, X - \tau(V + \frac{\hbar k}{2m}), p + \hbar k) - \\ & \left. - N_k D_0(t - \tau, X - \tau(V + \frac{\hbar k}{2m}), p)] \right\} , \end{aligned} \quad (57)$$

where $V = p/m$. Further standard assumptions about spatial dependence (i.e. X -dependence) of the density matrix and standard reasonings (or “conjurations”), along with formal identity

$$\lim_{t \rightarrow \infty} \sum_{\sigma} \int_0^t \exp[\sigma(\dots)\tau] d\tau = 2\pi \delta(\dots) ,$$

help to transform (57) into what is called “kinetic equation”,

$$\begin{aligned}
 \dot{D}_0(t, X, p) = & -V \nabla_X D_0(t, X, p) + \\
 & + \frac{2\pi}{\hbar^2} \int |c_k|^2 \left\{ \delta(\omega_k - kV + \frac{\hbar k^2}{2m}) \times \right. \\
 & \times [N_k D_0(t, X, p - \hbar k) - (N_k + 1) D_0(t, X, p)] + \\
 & + \delta(\omega_k - kV - \frac{\hbar k^2}{2m}) \times \\
 & \times [(N_k + 1) D_0(t, X, p + \hbar k) - N_k D_0(t, X, p)] \left. \right\} \frac{d^3 k}{(2\pi)^3} ,
 \end{aligned} \tag{58}$$

which is one-electron version of more general kinetic equation for electrons in vibrating crystal lattice [26].

Thus we have demonstrated reducibility of our exact evolution equations to standard kinetic models.

Unfortunately, “to break is not to make”! The truncation of hierarchy of electron-many-phonon correlations is much easier than revealing of their actual predestination and physical sense. Therefore let us go to summing up our actual results.

6. Discussion and resume

The kinetic equation (58) takes into account only simplest mutually non-correlated (mutually incoherent) electron transitions, each involving one phonon. In reality, however, electron’s interaction with any particular phonon mode k , - described by operators $a_k \exp[ikr(t)]$ and $a_k^\dagger \exp[-ikr(t)]$ in the Heisenberg representation over other modes, - depends on random electron’s path $r(t)$ determined by simultaneous interactions with all these modes. Hence, there are also various multi-stage electron’s transitions which involve mutually correlated (mutually coherent) acts of absorption or irradiation of two, three or more phonons. Hierarchy of the above introduced correlation functions Δ_n and D_n just describes how these many-particle processes “dress” the electron and constitute statistics of its random walk.

Analogous correlated many-particle process were considered in [27], where quantum amplitude of any particular electron transition was influenced by electric voltage fluctuations due to simultaneously realizing transitions of other electrons. There it was demonstrated that the correlated transitions are responsible for low-frequency (“quasi-static”, or 1/f-type) fluctuations of one-electron “transition probabilities” and consequently electric conductance.

Here, we performed similar but rigorous analysis of electron (quantum particle) interacting with infinitely many phonon modes (boson thermostat) and showed possibility of shortened but complete description of this system in terms of the mentioned electron-many-phonon correlation functions whose arguments include electron variables and phonon indices only.

Main formal result of the present paper is derivation of exact hierarchy of evolution equations for these correlation function and thus for electron’s density matrix. That are

Eqs.33 or Eqs.37 and their generating functional equivalent, Eq.40. For the best of my knowledge, such equations never earlier were in use.

It should be underlined that our recipe of exact shortened description can be extended to (generally relativistic) “dressed” electron in electromagnetic field and to many-electron (or, generally, many-fermion) systems.

The correlated many-particle processes (or “kinetic events”, or “dynamic clusters”, etc.) in classical statistical mechanics of fluids [1, 2, 3, 4, 5, 8, 9] and dielectric crystals [15] are under our attention after the work [7] where for the first time it was shown that a fluid particle has no definite “probability of collisions”, diffusivity and mobility. In other words, any of such characteristics of particle’s random motion undergoes scaleless $1/f$ -type low-frequency fluctuations. The theory discovers them when it does not neglect n -particle statistical correlations which can arise between particles participating in chains (clusters) of $n - 1$ dynamically connected consecutive collisions. Infiniteness of variety of such chains means originality of any phase trajectory of the system [4, 14] and impossibility of its fully adequate description in terms of two-particle events only, even in case of arbitrarily dilute gas [1, 3, 4, 5].

Close ideas concerning random motion of electrons (or holes) in electric conductors, - e.g. intrinsic or weakly doped semiconductors, - were formulated already in [10, 11, 28, 29] (for later formulations see also [12, 13]), with the aim to suggest principal explanation of electric $1/f$ -noise observed in various conductors [11] and usually manifesting itself as $1/f$ fluctuations in mobilities of charge carriers [11, 30, 31, 32]. The present work is first step from the phenomenological theory of electronic $1/f$ -noise [10, 11, 28, 29] to its microscopic statistical-mechanical theory.

In this respect, I would like to emphasize close conformity of the above obtained Eqs.37 and 40 for “Brownian particle” (electron) in ideal phonon gas and previously obtained evolution equations for many-particle correlations “dressing” a molecular Brownian particle in usual classical ideal gas [1, 3, 4, 5]. Solution of Eqs.37 or 40 can be represented by an (operator-valued and branching) infinite continued fraction similar to that in [1]. This conformity prompts that the Eqs.37 or, equivalently, 40 also hide $1/f$ fluctuations of diffusivity and mobility. Their visualization must be second step to the desired theory.

At that, of course, none truncation of Eqs.37 is appropriate. The generating-functional formulation, (40), of Eqs.37 make it obvious that all floors of the corresponding continued fraction, - i.e. all the electron- n -phonon irreducible correlations, - have equal rights and significance, since all they are represented in Eq.40 by only three simple terms. And it is so at arbitrarily weak electron-phonon coupling. Weaker coupling implies only that upper frequency bound of $1/f$ fluctuations of rate of electron relaxation becomes lower proportionally to the rate itself, and no more. Hence, one needs in adequate methods of operating with the whole electron’s correlation “coat”. This will be third step to valuable theory of electron-phonon interaction.

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